MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

EME4086 – FINITE ELEMENT METHOD
(ME)

27 OCTOBER 2017
3.00 p.m. - 5.00 p.m.
(2 Hours, Open Book)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 6 pages with 4 Questions and 1 Appendix only.
2. Attempt **ALL FOUR** questions of 25 marks each.
3. Please write all your answers in the Answer Booklet provided.

Question 1

Figure Q1 shows a system consisting of five members which is fixed at both far ends. Four concentrated forces are applied to the system. Young's modulus, E for all the members is 200 GPa. The applied forces (in Newton) and cross sectional area (in m^2) of the members are: $F_A = 3560$, $F_B = 4450$, $A_1 = 2 \times 10^{-4}$, $A_2 = 1 \times 10^{-4}$, $A_3 = 6 \times 10^{-4}$, $A_4 = 1.3 \times 10^{-4}$ and $A_5 = 1.3 \times 10^{-4}$. The lengths (in meters) are $L_A = 0.15$ and $L_B = 0.1$.

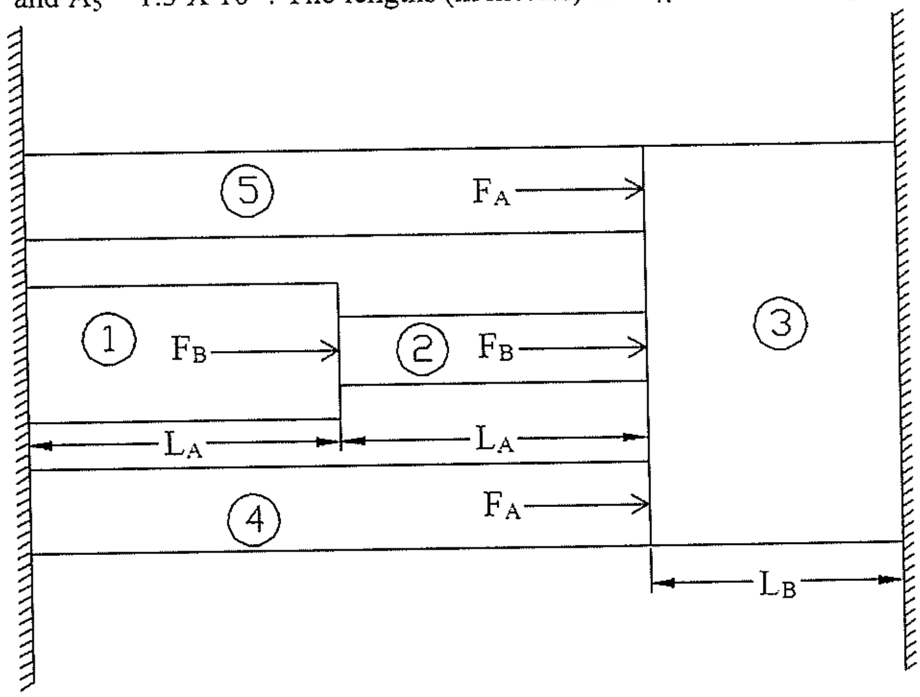

Figure Q1

a) Determine **all** the unknown forces and deflections for the system shown in **Figure Q1** by using minimum number of one-dimensional finite elements. Do not change the given element numbers. **[16 marks]**

b) Verify the unknown forces computed in part a) by using static equilibrium. **[2 marks]**

c) Compute the axial stress for each member and indicate whether it is tensile or compressive. **[5 marks]**

d) Without increasing the number of finite elements, suggest a method to determine deflection at the midpoint of member 2. **[2 marks]**

Continued ...

Question 2

Consider a nonlinear equation:

$$-\left(\frac{du}{dx}\right)^2 = \cos \pi x \qquad for\ 0 < x < 1$$

subjected to boundary conditions:

$$u(0) = 0 \qquad and \qquad u(1) = 0$$

Choose: $\phi_i = \sin(i\,\pi\, x)$ and do the following:

a) show, in detailed steps, that the weak form for the nonlinear equation above is given as:

$$B(v,u) = \int_0^1 \left(\frac{du}{dx}\right)\left(\frac{dv}{dx}\right) dx$$
$$l(v) = \int_0^1 v \cos(\pi x)\, dx$$
$$v = trial\ \ function$$

[10 marks]

b) find a two-parameter approximate solution by using the Ritz method.

[15 marks]

Hint: Please refer to the Appendix on page 6 to solve the integrations.

Continued ...

Question 3

a) For the truss shown in **Figure Q3 (a)**, given that each truss has a cross sectional area of 0.005 m^2, Young's modulus of 100 GPa and yield strength of 5.2 MPa. Without changing the given node numbers and element numbers, do the following:

i) determine the displacement at node 3. **[13 marks]**

ii) compute the internal force of each element of the plane truss. Indicate whether it is tensile or compressive. **[6 marks]**

iii) determine whether the applied loads are safe or not. **[2 marks]**

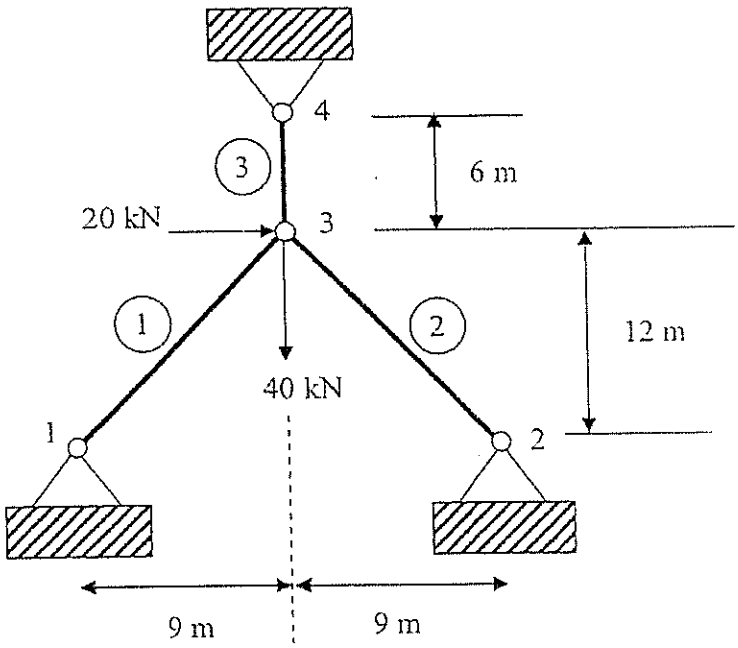

Figure Q3 (a)

b) Consider a single finite element with fixed cantilever support condition as shown in **Figure Q3 (b)**.

i) Are you able to obtain correct solution for the displacements at the tip by using a truss element? Explain. **[2 marks]**

ii) Propose other suitable finite element to do the task and justify your proposal. **[2 marks]**

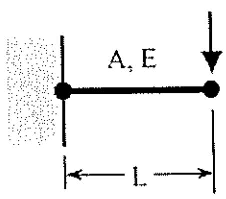

Figure Q3 (b)

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Question 4

A two-dimensional heat transfer finite element model is shown in **Figure Q4**. The plate is of a unit thickness, a unit length and a unit width. Thermal conductivity, $k_x = k_y = 1.0$ W/(m^2-K). The problem is symmetric about the axis going through Nodes 2 and 5. Node 3 is the midpoint. The right edge is maintained at $T_0 = 200^oC$, and the rest of the edges are maintained at $T = 0^oC$.

a) For the half-model, give $[K][T] = [Q]$, where $[K]$ is the global "stiffness" matrix, $[T]$ is the global primary variable vector, and $[Q]$ is load vector. **[15 marks]**

b) Determine the temperature at Node 3. Let $T_4 = 200\ ^0C$. **[7 marks]**

c) Compare the temperature computed at Node 3 in part **b)** and the one computed from the first three terms of the analytical solution given below. Comment on the differences, if there are any.

$$T = T_0 \frac{2}{\pi} \sum_{n=1,3,5}^{\infty} \left(\frac{(-1)^{n+1}+1}{n} \right) \left(\frac{\sinh(n\pi x)}{\sinh(n\pi)} \right) \left(\sin(n\pi y) \right)$$

[3 marks]

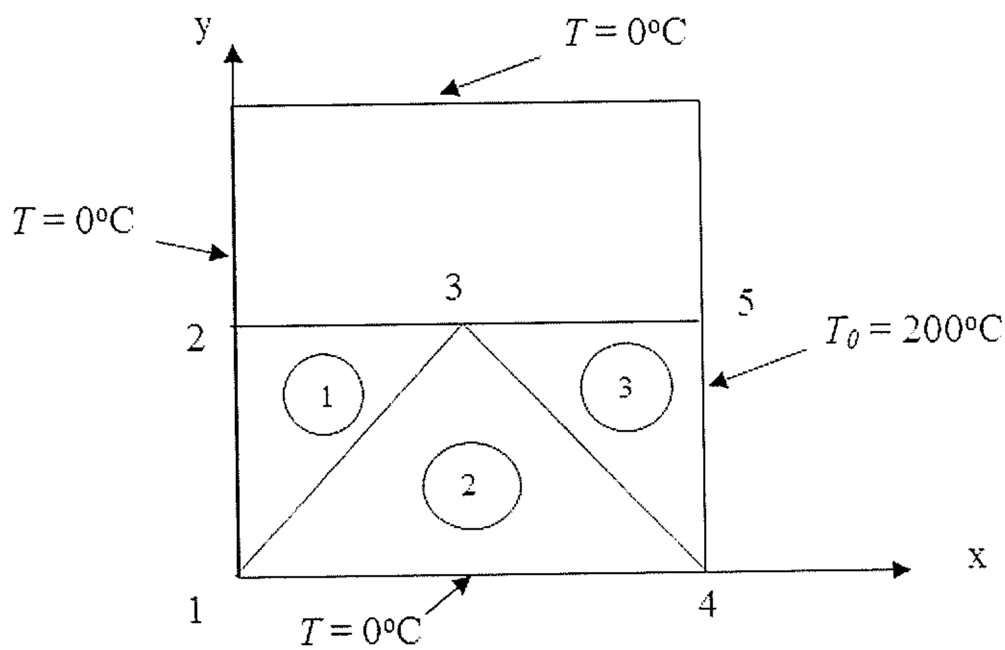

Figure Q4

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Appendix

$$\frac{d}{dx} Sin(i\pi\, x) = i\pi\, Cos\left(i\pi\, x\right)$$
$$\frac{d}{dx} Cos(i\pi\, x) = -i\pi\, Sin\left(i\pi\, x\right)$$

$$\int_0^1 Cos(i\pi\, x)\, Cos(\, j\pi\, x)\ dx = \begin{cases} 0 & i \neq j \\ \frac{1}{2} & i = j \end{cases}$$

$$\int_0^1 Sin(i\pi\, x)\, Cos(\pi\, x)\ dx = \begin{cases} 0 & i \text{ is odd} \\ \dfrac{2i}{\pi\left(i^2 - 1\right)} & i \text{ is even} \end{cases}$$

$$Integration\ by\ parts : \int_a^b w\ dz \;=\; wz\Big|_a^b - \int_a^b z\ dw$$

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